

Kaufman - Trading Systems & Methods - Chap08

Chap08

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8 Cycle Analysis

The cycle is another basic element of price movement, along with the trend and seasonality, but it is more difficult to evaluate; therefore, it is often overlooked. Cycles come in many forms—seasonality, production startup and shutdown, inventory or stocks, behavioral, and astronomical. Seasonality is a special case of a calendar or annual cycle. Seasonality was covered in the previous chapter and its special features are not considered here. Some of the cycles are clearly periodic, having regular intervals between peaks and valleys; others are more uniform in their amplitude or height but irregular in period. The most definitive and regular cycle remains the seasonal, which is determined by periodic physical phenomena.

This chapter will discuss the major commodity cycles that result from business decisions, government programs, longterm market characteristics, and phenomena. Shortterm cycles are usually attributed to behavior and will be covered in a later chapter on pattern recognition. There are a few important ways to find the cycle, the most common being trigonometric curve fitting and Fourier (spectral) analysis. Both will require a computer and will be explained in the following sections. Examples of solutions will be included in the explanation of the methods and applications will follow. Computer programs that solve the trigonometric problems can be found in Appendix 4 along with additional examples, CYCLE BASICS

Even when the seasonal pattern is eliminated, most cycles are still based on the periodic effects in our solar system. After the 1year orbit of our planet around the sun, there is the 28day lunar cycle; converted to business days, this gives the very familiar 20day reference, which remains overwhelmingly popular among all analysts. Other planetary effects, which should by no means be discarded offhand, can be found in Chapter 14 under the topic "Financial Astrology." Throughout this chapter, there should be the undercurrent that planetary motion may account for the consistency of any cycle that repeats with a fixed period.

Cycles can be complex and difficult to see because there are often larger and smaller patterns, and cycles within cycles, all acting at the same time. Nevertheless, they exist and they are real. The cycles that appear to be most important are either longterm or the sum of a number of subcycles that come together at peaks or bottoms. This gives us a way to identify one point on a cycle; we must remember that, when the individual components are found, there may be a number of smaller patterns that cause this effect. A reference to harmonics, just as in music, means that a smaller cycle is a fraction of the larger (for example, its cycle length is $1/2 K$, $1/4$ of the larger). When two cycles are synchronized, their peaks or valleys occur at the same time. Any price series can be decomposed into individual cycles, although there may be many cycles needed.

Observing the Cycle

Before selecting a market for cycle analysis, it is necessary to observe that a dominant Cycle exists; it is also useful to know why it exists to avoid uncovering spurious patterns. This is

most easily done for markets in which you can clearly identify the fundamental and/or industrial reasons for cycles. The basis for a cycle could be a pattern of holding inventory, the fixed time needed for breeding and feeding of livestock, seasonality or other economic factors. Figure 81 shows a clear 9 to 11 month cycle in live cattle futures prices' over a past 6 year period. The peaks and valleys vary by up to 1 month, making the pattern reliable for use as part of a longterm trading strategy.

The Swiss franc cycle demonstrated in Figure 82 is quite different.' It ranges from 24 to 35 weeks with a 40% variance compared with 20% for cattle. Most important, the cycle in the Swiss franc cannot be related to any specific fundamental reason. Given enough variance, a repeating pattern can be found in any data. To rationally trade a cyclic pattern, there must be a reason for its existence and the confidence that it should continue.

A simple way to begin the search for major cycles is to look at a longterm chart, displayed as weekly rather than daily prices. The dominant halfcycle can be found by locating the obvious price peaks and valleys, then averaging the distance between them. A convenient tool for estimating the cycle length is the Ehrlich Cycle Finder. 3 It is an expanding device with evenly spaced points, allowing you to align the peaks and valleys to observe the consistency in the cycle. For finding a single pattern, it is just as good as some of the mathematical methods that follow. It is best to have at least eight cycle repetitions before concluding that you have a valid cycle.

Cycles can be obscured by other price patterns. A strong trend, such as the one in Swiss francs (Figure 82) or the seasonal movement of crops, may overwhelm a less pronounced cycle. Proper cycle identification requires that these factors first be removed by detrending and by deseasonalizing. The resulting data will then be analyzed and the trend and seasonal factors added back once the cycle has been found. To find a subcycle, the primary cycle should be removed and a second cycle analysis performed on the data. The methods that follow (trigonometric regression and spectral analysis) locate the dominant cycles at one time using integrated processes.

The Kondratieff Wave

Much of the popularity of cycles is due to the publicity of Nicolai Kondratieff's 54year cycle, known as the Kwave. During its documented span from about 1780 to the present, it appears to be very regular, moving from highs to lows and back again. In Figure 83 the Kondratieff wave is shown with the Wholesale Price Index as a measure of economic health. 4 With only three full cycles completed, it is difficult to tell if the overall trend is moving upward or whether the entire pattern is just a coincidence.

The forecast of the Kwave, shown in Figure 83, indicates a sharp decline in wholesale prices due at about the year 1990, the millenium's equivalent to the depression of the 1930s. In fact, the early 1990s have posted remarkable gains in the stock market, and at the end of the 1990s there is talk of disinflation from the U.S. Federal Reserve; this may serve as some support for a continued cycle. Although interesting, a cycle of this

length is impractical other than to point out that the peaks and valleys could vary by as much as 10% from the predicted values without diminishing the validity of the theory For the Kondratieff wave, a shift of 10%, as seen in the 1930s, would cause a trading signal to buy 5 years too soon or too late. Determining cycles for any market has the same problemthe actual

Jacob Bernstein, "Cyclic and Seasonal Price Tendencies in Meat and Livestock Markets, in Trading Tactics: A Livestock Futures Anthology, Todd Lofton (ed.) (Chicago Mercantile Exchange, Chicago, 1986).

Jacob Bernstein, The Handbook of Commodity Cycles (John Wiley & Sons, New York, 1982).

1 Elirlich Cycle Finder Company, 2220 Noyes Street, Evanston, IL 60201.

Jeff Walker, "What Kwave?," TecAmical Analysis of Stocks & Commodities (July 1990).

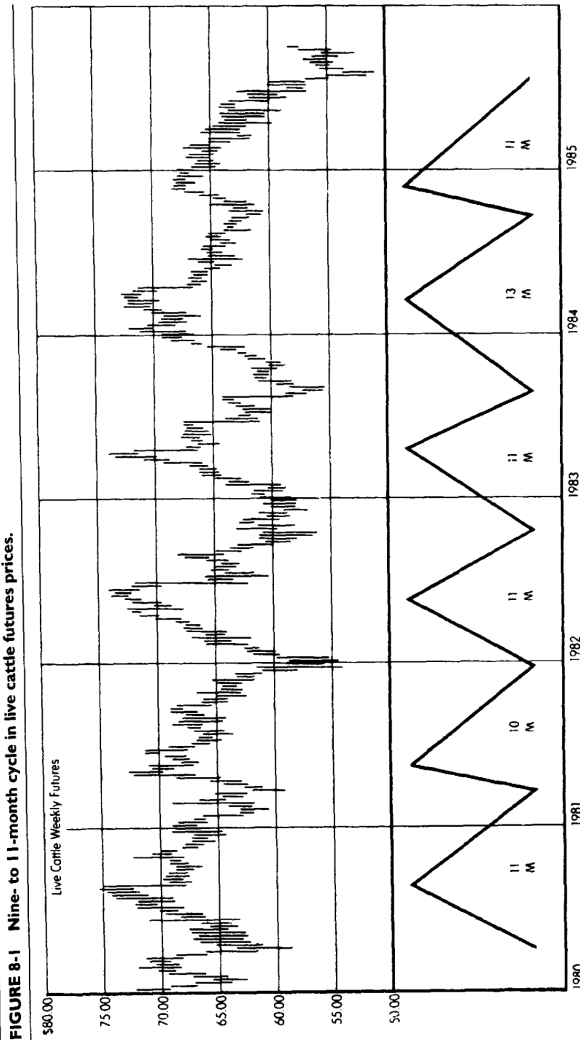
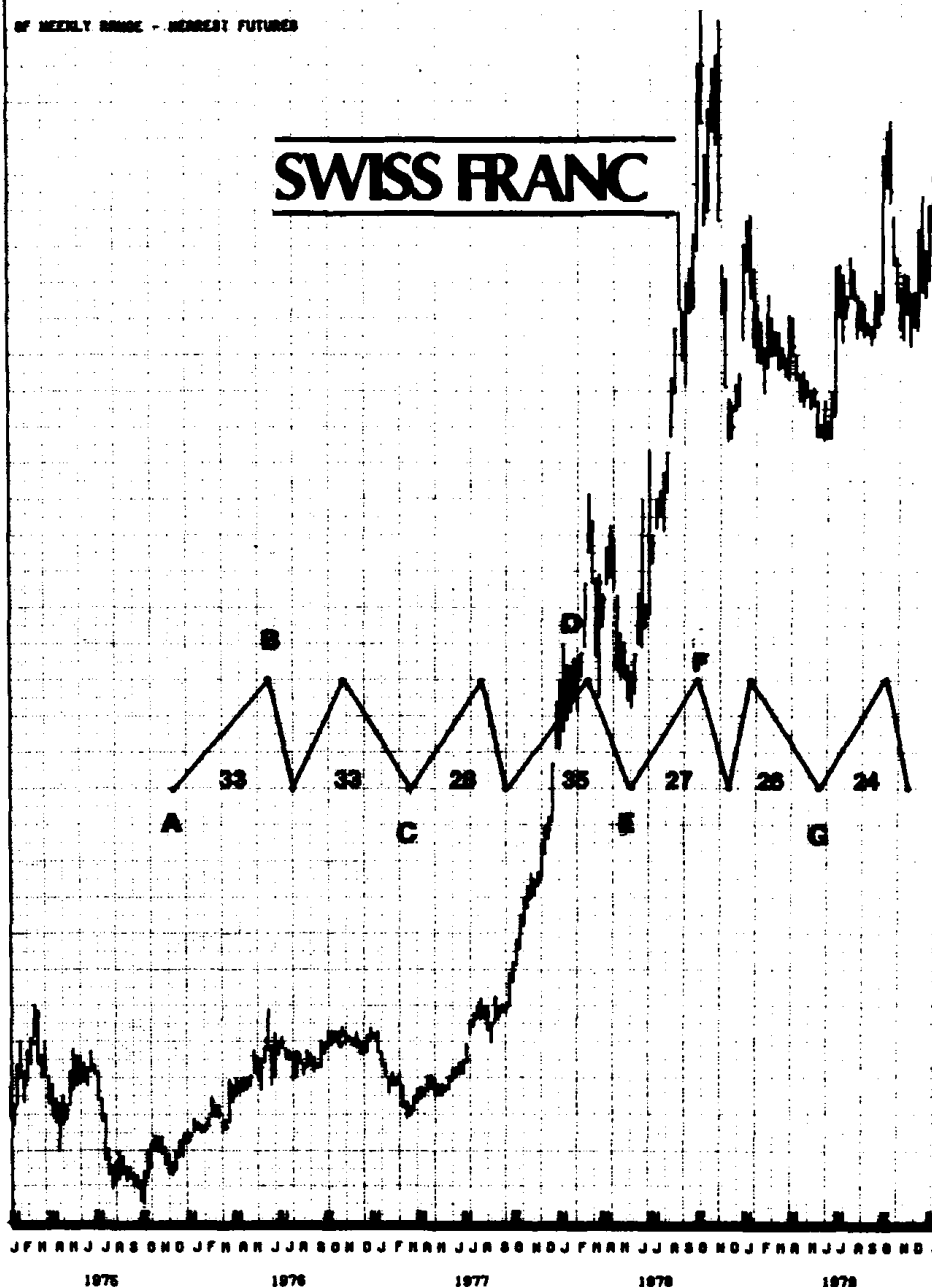


FIGURE 8-1 Nine- to 11-month cycle in live cattle futures prices.

FIGURE 82 Cycles in Swiss franc futures.

OF WEEKLY RANGE - NEAREST FUTURES

SWISS FRANC



Source: J. Bernstein, *The Handbook of Commodity Cycles* (New York: John Wiley & Sons, 1982).
 price pattern will never correspond exactly to the predicted peaks and

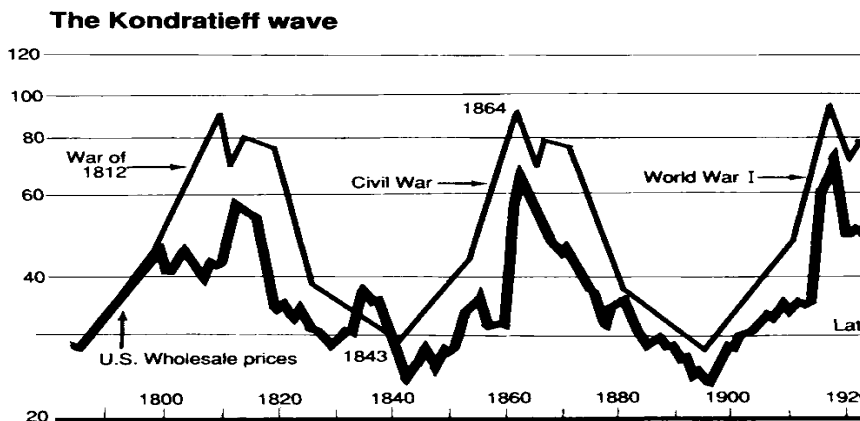
valleys, which, because they are a mathematical cycle, must come at regular intervals.

Terminology

Before getting very technical about the measurement and calculation of cycles, there are a few terms that describe most of the concepts discussed throughout this chapter. Note that the use of wave and cycle is interchangeable.

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FIGURE 83 The Kondratieff Wave.



Source: Jeff Walker, "What K-wave?" *Technical Analysis of Stocks & Commodities*, Technical Analysis, Inc. Used with permission.

Amplitude the height of the wave (cycle)

Period the time needed to finish one wave (one cycle)

Frequency (for the more mathematical) the number of cycles that repeat every 3600

Phase a measurement of the separation of tops or bottoms of two waves with the same period

Left and right translation the tendency for a cycle peak to fall to the left or right of the center of the cycle

UNCOVERING THE CYCLE

Before resorting to the highly mathematical methods for finding cycles, there are some simple approaches that may serve many traders. For example, if you believe that there is a dominant 20day cycle, then you simply create a new price series by subtracting the current data from a 20day moving average. This removes the trend that may obscure the cycle. Most oscillators, such as a stochastic or RSI, also serve to identify a price cycle.

Enhancing the Cycle by Removing the Trend

The cycle can become more obvious by removing the price trend. The use of two trendlines seems to work very well in most cases. (5) First, smooth the data using two exponential moving averages, in which the longer average is half the period of the dominant cycle (using your best guess), and the shorter one is half the period of the other. Then create an

5 In his article, "Finding Cycles in Time Series Data (Technical Analysis of Stocks & Commodities) (August 1990)), A. Bruce Johnson credits John Ehlers for his work in the use of two exponential trends. See John Ehlers, "Moving Averages, Part 1" and "Moving Averages, Part 2," Technical Analysis of Stocks & Commodities (1988).

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MACD index by subtracting the value of one exponential trend from the other., the resulting synthetic series avoids the lag inherent in most methods.

A technique that enhances the cycle is the use of triangular weighting instead of exponential smoothing. This method creates a set of weighting factors that are smallest at the ends and peak in the middle of the weighted average. For example, if you wanted a 10-period triangular average, the peak weighting factor would always be 2.0 and is assigned to the price just before the middle (there is no middle value for an even number of periods; therefore, the first value is eliminated), then weighting factors would begin and increase by $2.0/(P/2)$, where $P = 10$. This creates the series of weighting factors, beginning at $t = P + 2$ (to get a center value you need to have an odd number of entries), $A, .8, 1.2, 1.6, 2.0, 1.6, 1.2, .8, .4$. Once the weighting factors, w , are known, the sum of the weighting factors is P (the same as the period), and the triangular average is

$$TMA_t = (W_1 \times P_{t-P+2} + W_2 \times P_{t-P+3} + \dots + W_{P-1} \times P_t)$$

When applying a triangular average, each of the two average (period of the other) get triangular weights and the difference of the triangular MACD in Figure 8-4 shows what appears to be the price of IBM.

Trigonometric Price Analysis

Most cycles can be found using the trigonometric function. These functions are also called *periodic waves*, because they repeat every 2π (approximately $= 3.141592$). Because radians can be converted to degrees

$$1 \text{ degree} = \frac{2\pi}{360}$$

all work that follows will be in degrees. Some other necessary terms

Amplitude (a)—the height of the wave from the center ($x = a$)

Frequency (ω)—the number of wavelengths that repeat each unit of time
 $\omega = 1/T$

Period (T)—the number of time units necessary to complete one cycle

A simple sine wave fluctuates back and forth from $+1$ to -1 in one cycle (one wavelength) as the degrees increase from 0° to 360° (or 2π radians). If you divide the wavelength to a specific distance in boxes (on graph paper), you get the number of boxes in a full wavelength, resulting in the box frequency. For example, a 100-box cycle would give a value of 3.6° to each box. You can change this to other than 360° by using the frequency as a multiplier for the angle of the wave.

$$\sin \omega\phi$$

If $\omega > 1$, the frequency increases and the wavelength shortens. If $\omega < 1$, the frequency decreases and the wavelength increases. Because ω gives the number of wavelengths in each 360° cycle. To change the starting point, the value b is added to the angle.

$$\sin (\omega\phi + b)$$

If b is 180° , the sine wave will start in the second half of the cycle. The **amplitude** can be changed by multiplying the sine wave by a constant.

There are few examples of price movement that can be represented by a single wave; thus, two sine waves must be added together to form a compound wave:

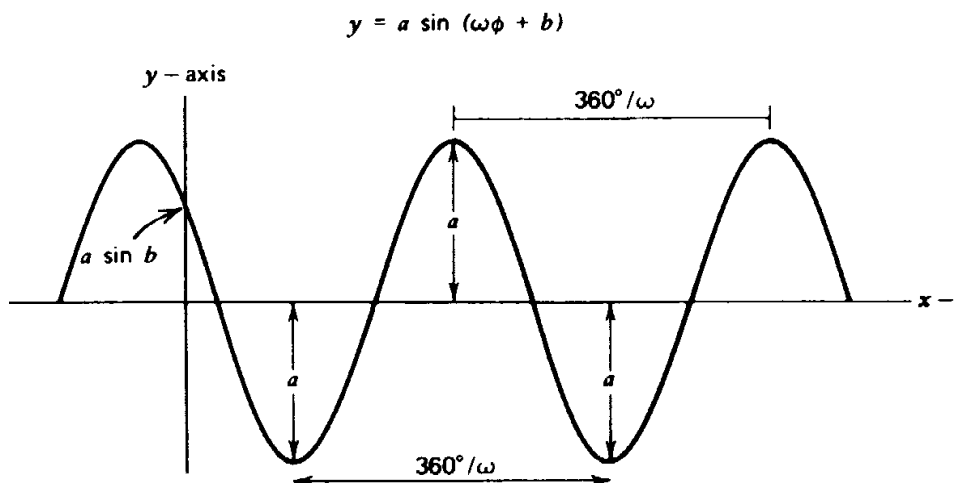
$$y = a_1 \sin (\omega_1 \phi + b_1) + a_2 \sin (\omega_2 \phi + b_2)$$

Each set of characteristic variables, a_1 , ω_1 , b_1 , and waves are measured at the same point ϕ at the same the phase constants b_1 and b_2 be zero:

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FIGURE 8-5 Sinusoidal wave.

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$$y_1 = 3 \sin 4\phi$$

$$y_2 = 5 \sin 6\phi$$

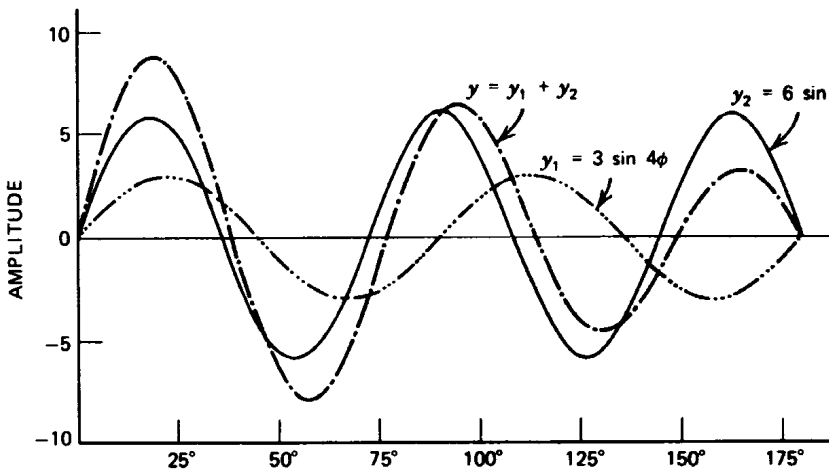
$$y = y_1 + y_2$$

Figure 8-6 shows the individual regular waves y_1 and y_2 , and the interval 0° to 180° . Note that both y_1 and y_2 began the next cycle by 180° , they are perfectly out-of-phase. During the next 180° they are back into phase.

When combining periodic waves, it is useful to know the maximum amplitude of the resulting wave. Because the peaks of the two waves do not necessarily fall at the same point, the maximum amplitude is not necessarily reached. A mathematical technique, called *differentiation*, is used to find the maximum and minimum amplitudes. The first derivative, with respect to ϕ , where y is the formula to be differentiated. The rules are:

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FIGURE 8-6 Compound sine wave.



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$$\frac{d}{d\phi} (\sin \phi) = \cos \phi; \quad \frac{d}{d\phi} (\cos \phi) = -\sin \phi$$

$$\frac{d}{d\phi} (\sin \omega\phi) = \omega \cos \omega\phi$$

$$\frac{d}{d\phi} (\sin (\omega\phi + b)) = \omega \cos (\omega\phi + b)$$

$$\begin{aligned} \frac{d}{d\phi} (a_1 \sin (\omega_1\phi + b_1) + a_2 \sin (\omega_2\phi + b_2)) \\ = a_1\omega_1 \cos (\omega_1\phi + b_1) + a_2\omega_2 \cos (\omega_2\phi + b_2) \end{aligned}$$

Applying this method to the previous example,

$$y = 3 \sin 4\phi + 6 \sin 5\phi$$

$$\frac{dy}{d\phi} = y' = 12 \cos 4\phi + 30 \cos 5\phi$$

The points of maximum and/or minimum value occur where the maxima and minima occur when $4\phi = 90^\circ$ and 270° ($\phi = 22\frac{1}{2}^\circ$ and $67\frac{1}{2}^\circ$). For $y'_2 = 30 \cos 5\phi$, the maximum and minimum values occur at $22\frac{1}{2}^\circ$ and $67\frac{1}{2}^\circ$ (and 54°). It must be pointed out that the first derivative identifies extreme highs and lows, but does not tell which one is the maximum. The second derivative, y'' , calculated by taking the derivative, has the purpose as follows:

If $y'(x) = 0$ and $y''(x) > 0$, then $y(x)$ is a minimum.

If $y'(x) = 0$ and $y''(x) < 0$, then $y(x)$ is a maximum.

Then, $y_1 = 22\frac{1}{2}^\circ$ and $y_2 = 18^\circ$ are maxima and $y_1 = 67\frac{1}{2}^\circ$ and $y_2 = 54^\circ$ are minima.

Anyone interested in pursuing the analysis of extrema will find many discussions in a text on calculus. Rather than concentrating on trigonometric curves,⁶ consider a practical example of finding a cycle in the price of copper in Table 8-1 and charted in Figure 8-7. The price peaks seem to occur in mid-1966, January 1970, and January 1974, about 4 years apart. The problems are tedious; therefore, calculations will be performed by computer programs in Appendix 4.

The results obtained by using actual copper prices will not be presented here. It is important to be able to understand the significance of the results and apply them effectively.

Because trigonometric curves fluctuate above and below zero, the first step is to detrend the data using the least-squares method. The equation for a straight line representing the upward bias of the data is then subtracted from the original data to detrend the data so they vary equally above and below the line from positive to negative values.

The straight line $y = a + bx$, which best represents the trend of the data, is found by the least-squares equations:

⁶ A more specific presentation of trigonometric curve fitting can be found in *Investment Analysis* (Prentice-Hall, 1976, Chapter 8). The material covered in this book is more general.

New York*

Average Quarterly Price (¢/lb)

Year	1st	2nd	3rd
1963	22.12	22.46	22.17
1964	23.18	24.56	25.57
1965	28.23	33.77	35.90
1966	46.22	51.48	40.76
1967	36.51	29.30	30.36
1968	39.75	30.07	29.08
1969	38.94	42.95	43.38
1970	47.70	46.98	35.78
1971	25.40	29.45	27.15
1972	32.74	33.53	30.01
1973	36.82	45.07	55.13
1974	66.56	70.06	47.30
1975	32.06	31.46	35.75
1976	38.22	43.24	45.46
1977	37.08	38.72	34.01
1978	35.07	40.23	41.63
1979	51.12	63.71	59.56

*Based on prices from the American Metal Market.

$$b = \frac{N \sum xy - \sum x \sum y}{N \sum x^2 - \left(\sum x \right)^2}$$

$$a = \frac{1}{N} \left(\sum y - b \sum x \right)$$

To do this, let x be the date and y be the price on that date. For the first year, let $x = 1967$, $1967\frac{1}{4}$, $1967\frac{1}{2}$, . . . , let $x = 1, 2, 3, \dots$. The solution program in Appendix 2 (also an integral part of the program for the hand-calculation method, is

$$y = 28.89 + .267x$$

Figure 8-7 displays the original copper prices and the regression line. The data can now be detrended using the equation above, subtracting the predicted prices from the responding prices. Complete step-by-step results for this example are in Appendix 4. The detrended data is now used in the general trigonometric wave:

$$y_t = a \cos \omega t + b \sin \omega t$$

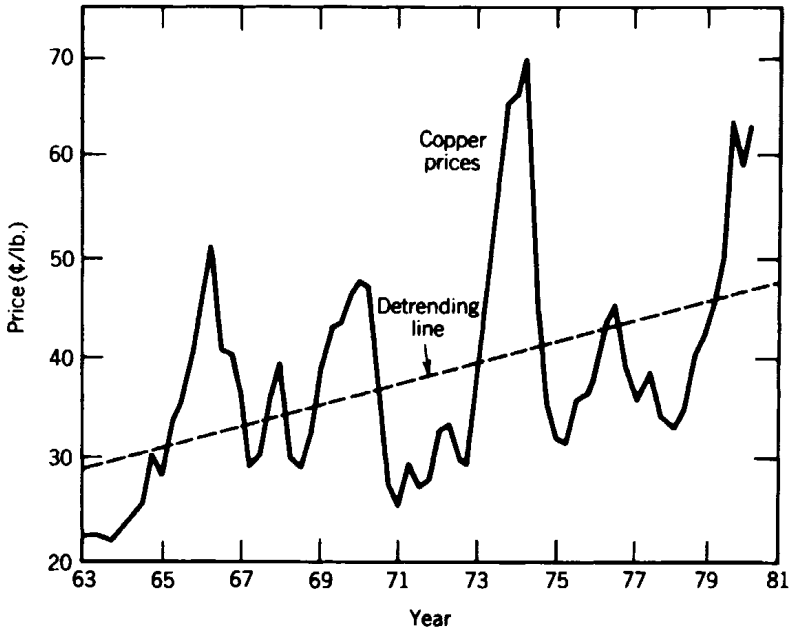
The variable t replaces ϕ to express the angle in integer units. The data will be more convenient to visualize and to chart.

To find the frequency ω , it will be necessary to first solve

$$\cos \omega - \frac{1}{2} \alpha = 0$$

using the system of equations,

(No. 2 heavy scrap)



$$\alpha y_2 = y_1 + y_3$$

$$\alpha y_3 = y_2 + y_4$$

$$\vdots$$

$$\alpha y_{n-1} = y_{n-2} + y_n$$

This is expressed as a summation (similar to least-squares) in which must be found:

$$\alpha \sum c^2 = \sum cd$$

where $c = y_n$ and $d = y_{n-1} + y_{n+1}$. Summing the detrended values 6338.4 and $\sum cd = 9282.2$, resulting in $\alpha = 1.464$. The value for α is substituted in the intermediate equation and solved for the frequency ω :

$$\cos \omega - \frac{1}{2} (1.464) = 0$$

$$\cos \omega = .732$$

$$\omega = 42.9$$

The period T is $360/42.9 = 8.4$ calendar quarters. The last step in the procedure for a single frequency is to write the normal equations:

$$a \sum \cos^2 \omega t + b \sum \cos \omega t \sin \omega t = \sum y_t \cos \omega t$$

$$a \sum \sin \omega t \cos \omega t + b \sum \sin^2 \omega t = \sum y_t \sin \omega t$$

and solve for a and b , where $t = 1, \dots, 40$, and $\omega = 42.9$. As in the computer program is best for finding the sums (using detrended data) equations. The sums are

$$a \sum \cos^2 \omega t \quad \sum \sin \omega t \cos \omega t \quad \sum y_t \cos \omega t$$

$$\sum \sin^2 \omega t \quad \sum y_t \sin \omega t$$

Then, a and b can be found by substituting in the following equations

$$b = \frac{\sum y_t \sin \omega t \sum \cos^2 \omega t - \sum y_t \cos \omega t \sum \sin^2 \omega t}{\sum \cos^2 \omega t \sum \sin^2 \omega t - \sum \cos \omega t \sin \omega t}$$

$$a = \frac{\sum y_t \cos \omega t - b \sum \cos \omega t \sin \omega t}{\sum \cos^2 \omega t}$$

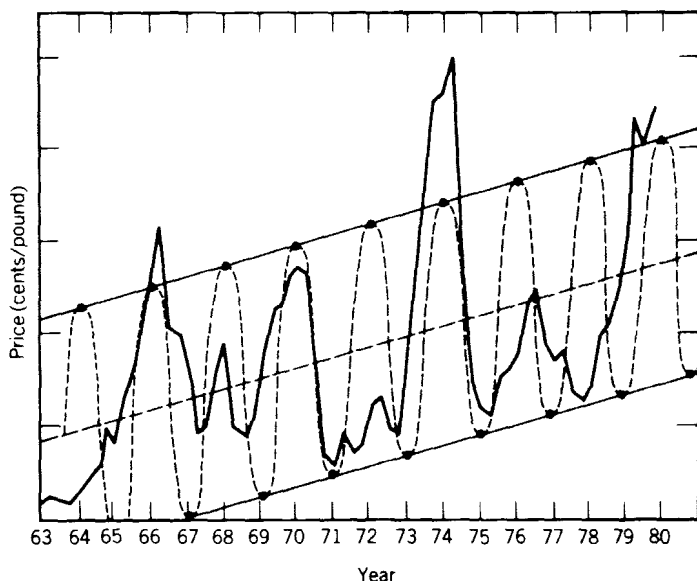
The results $a = -.603$ and $b = 1.831$ give the single-frequency curve

$$y_t = -.603 \cos 42.9t + 1.831 \sin 42.1t$$

Taking $t = 1$ to be 1967 and $t = 68$ to be 1979½ and adding back the periodic curve is shown in Figure 8-8.

The single-frequency curve shown in Figure 8-8 matches several in copper; however, it is not much more than could have been

FIGURE 88 Copper prices 1963-1979: single-frequency copper cycle manually scaled to approximate amplitude.



Copper prices 1963-1979

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Cycle Finder mentioned in a previous section. A singlefrequency curve can be created simply by identifying the most dominant peaks, averaging the distance (period), and applying the singlefrequency formula.

TwoFrequency Trigonometric Regression

The combination of more than one set of sine and cosine waves of varying amplitudes and frequencies will create a better fit than a singlefrequency solution. This is analogous to the use of a secondorder (curvilinear) solution instead of the firstorder linear. The equation for the twofrequency cycle is

$$y_t = a_1 \cos \omega_1 t + b_1 \sin \omega_1 t +$$

To find the results of this complex wave, apply the same techniques used in the singlefrequency approach to the detrended copper data. The algebra for solving this problem is an expanded form of the previous solution, and the use of a computer is a requirement. The programs necessary to solve this one appear in Appendix 4. The frequencies 0), and 0), are found by solving the quadratic equation:

$$2x^2 - \alpha_1 x - (1 + \alpha_2/2) = 0$$

where $x = \cos \omega$, using the standard formula:

$$x = \frac{\alpha_1 \pm \sqrt{\alpha_1^2 + 8(1 + \alpha_2/2)}}{4}$$

The same least-squares method as before can be used, derived from

$$\alpha_1(y_n + y_{n+2}) + \alpha_2 y_{n+1} = y_{n-1} + y_{n+3}$$

The least-squares equations for finding α_1 and α_2 are:

$$\alpha_1 \sum c^2 + \alpha_2 \sum cd = \sum cp$$

$$\alpha_1 \sum cd + \alpha_2 \sum d^2 = \sum dp$$

where $c = y_n + y_{n+2}$, $d = y_{n+1}$, and $p = y_{n-1} + y_{n+3}$. These equations can be solved for α_1 and α_2 using:

$$\alpha_2 = \frac{\sum dp \sum c^2 - \sum cp \sum cd}{\sum d^2 \sum c^2 - \left(\sum cd\right)^2}$$

$$\alpha_1 = \frac{\sum cp - \alpha_2 \sum cd}{\sum c^2}$$

Then, ω_1 and ω_2 are calculated from the two solutions x_1 and x_2 . The next step is to solve the normal equations to find the amplitudes

$$a_1 \sum \cos^2 \omega_1 t + b_1 \sum \cos \omega_1 t \sin \omega_1 t + a_2 \sum \cos \omega_1 t \cos \omega_2 t + b_2 \sum \cos \omega_1 t \sin \omega_2 t = \sum y_t \cos \omega_1 t$$

$$\begin{aligned}
& a_1 \sum \sin \omega_1 t \cos \omega_1 t + b_1 \sum \sin^2 \omega_1 t + a_2 \sum \sin \omega_1 t \cos \omega_2 t \\
& + b_2 \sum \sin \omega_1 t \sin \omega_2 t = \sum y_i \sin \omega_1 t \\
& a_1 \sum \cos \omega_2 t \cos \omega_1 t + b_1 \sum \cos \omega_2 t \sin \omega_1 t + a_2 \sum \cos^2 \omega_2 t \\
& + b_2 \sum \cos \omega_2 t \sin \omega_2 t = \sum y_i \cos \omega_2 t \\
& a_1 \sum \sin \omega_2 t \cos \omega_1 t + b_1 \sum \sin \omega_2 t \sin \omega_1 t + a_2 \sum \sin \omega_2 t \cos \omega_2 t \\
& + b_2 \sum \sin^2 \omega_2 t = \sum y_i \sin \omega_2 t
\end{aligned}$$

Once the sums are obtained, the final step is to create a 4×5 matrix to solve the four normal equations for the coefficients a_1 , b_1 , a_2 , and b_2 . When plotting the answer it will be best to plot the original two-frequency equation in its component forms as well as in combination:

$$y_i' = a_1 \cos \omega_1 t + b_1 \sin \omega_1 t$$

$$y_i'' = a_2 \cos \omega_2 t + b_2 \sin \omega_2 t$$

$$y_i = y_i' + y_i''$$

where $a_1 = 3.635$, $b_1 = -.317$, $a_2 = -.930$, and $b_2 = .762$. The solution to the two-frequency problem gives the following values:

$$\alpha_1 = .535 \quad x_1 = .830$$

and

$$\alpha_2 = .133 \quad x_2 = -.764$$

and finally the frequencies:

$$\omega_1 = 33.9 \quad \text{and} \quad \omega_2 = 139.8$$

correspond to 10.6 and 2.6 calendar quarters (Figure 8-9).

Fourier Analysis: Complex Trigonometric Regression

Developed by the French mathematician John Baptiste Joseph Fourier, *Fourier analysis* is a method of complex trigonometric regression, which expresses any data set as a series of sine and cosine waves of the same type as discussed in the previous section.

Assuming that there is a cycle and that there are N data points in each repetition of this cycle, the Fourier method of analysis shows that the N points lie on the regression curve:

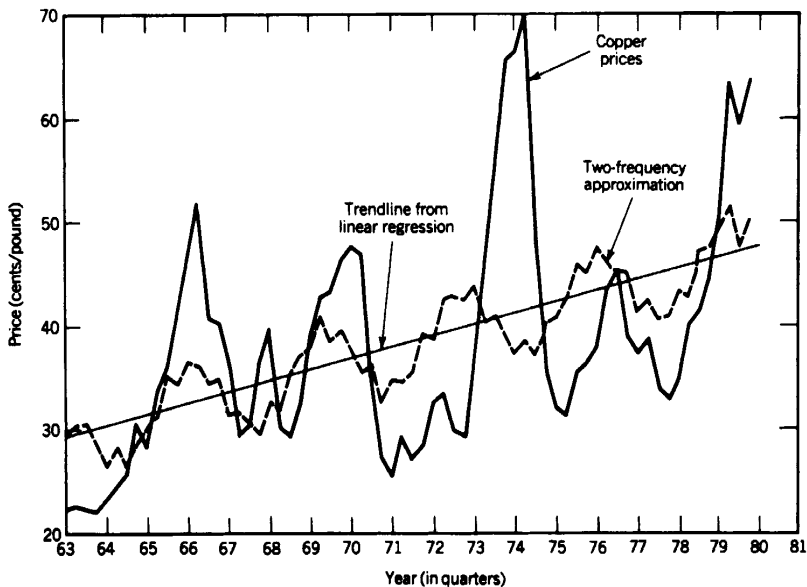
$$y_i = 1 + \sum_{k=1}^{(N/2)} \left(u_k \cos \frac{2\pi ki}{(N/2)} + v_k \sin \frac{2\pi ki}{(N/2)} \right)$$

where the regression coefficients u_k and v_k are given by:

$$u_k = \frac{1}{(N/2)} \sum_{i=1}^N y_i \cos \frac{2\pi ki}{(N/2)} \quad k = 1, 2, \dots, \frac{N}{2}$$

$$v_k = \frac{1}{(N/2)} \sum_{i=1}^N y_i \sin \frac{2\pi ki}{(N/2)} \quad k = 1, 2, \dots, \frac{N}{2}$$

$$v_{N/2} = 0$$



It is important to see that the mean of all the points on one cycle of y_i will have the property

$$\sum_{i=1}^N y_i = N$$

Applying the Fourier series to the seasonal component will help clarify this method. Seasonal data form the most obvious cycle. Using average monthly prices, detrended to avoid letting the trend overwhelm the cycle, let $N = 12$. It is also known that seasonally adjusted prices will vary about the mean; hence, the weighting factors will have the same property as the above equation. With this information, the trigonometric curve, which approximates the seasonals, can be generated and compared with the results of other methods.'

Spectral Analysis

Derived from the word spectrum, spectral analysis is a statistical procedure that isolates and measures the cycles within a data series. The specific technique used is the Fourier series as previously discussed, although other series have also been used.

When studying the cycles that comprise a data series, it is important to refer to their phase with respect to each other. Phase is the relationship of the starting points of different cycles. For example, if one cycle has the same period as another but its peaks and valleys

7 A continuation of this development can be found in Warren Gilchrist, *Statistical Forecasting* (John Wiley & Sons, London, 1976, pp. 139-148); a more theoretical approach is to be found in C. Chatfield, *The Analysis of a*

leys are exactly opposite, it is 180' outofphase. if the two cycles are identical in phase, they are coincident. Cycles with the same period may lead or lag the other by being outofphase to various degrees.

A tool used in spectral analysis to visualize the relative significance of a series' cyclic components is the periodogram. Weighting the cyclic components in the periodogram will give the more popular spectral density diagram, which will be used to illustrate the results of the spectral analysis. Density refers to the frequency of occurrence. Figures 810a and 810b show the spectral density of a series composed of three simple waves (D is the

Fourier series made up of waves A, B, and C).⁸ The cycle spectral density chart, corresponds exactly to the cycle B, and C. The spectral density, measured along the left amplitude squared of the cycle and the magnitude of the components, which obscures the cycle. In Figure 8-10b, the noise of only three pure waves. Had there been noise of the cycle amplitude, those cycles identified by the spectral density would be completely obscured. Readers who have studied ARIMA will find the spectral density and the correlogram.

As in trigonometric regression analysis, the other basic price components can distort the results. A noticeable trend in the data must be removed or it will be interpreted as the dominant cycle. The familiar methods of first differencing or linear regression can be used to accomplish this. The seasonal component is itself a cycle and does not need to be removed from the series. Because spectral analysis will identify both the seasonal and cyclic components, the success of the results will depend on the strength of these waves compared with the noise that remains. In applying this technique to real data, it would not be surprising to see the results demonstrated in Figure 811. Three subcycles of length 10, 20, and 40 days are shown as part of a 250day (seasonal) cycle. Notice that, as the cycle lengthens, the width of the spectral density representation widens. This does not mean that the wider peaks are more important.

The trader is most interested in those cycles with greater spectral density, corresponding to a larger price move. The minimum amount of data necessary to find these cycles must include the full cycle that might be identified. For example, to see any seasonal pattern, a minimum of 12 months is needed. More data is better when using spectral analysis to confirm the consistency of the cycle. A single year is not adequate to support any seasonal findings.

Weighting Factors

The most important part of spectral analysis is finding the proper estimators, or weighting factors, for the single-frequency series of cosine waves. When looking for long-term cycles, it is worth being reminded that the trend and seasonal components must be removed, because the method of spectral analysis will consider these the dominant characteristics, and other cycles may be obscured.

As in the other trigonometric formulas, the $t = 1, 2, \dots, N$ are the data points and $\hat{y}_t$ will be the spectral analysis. Then

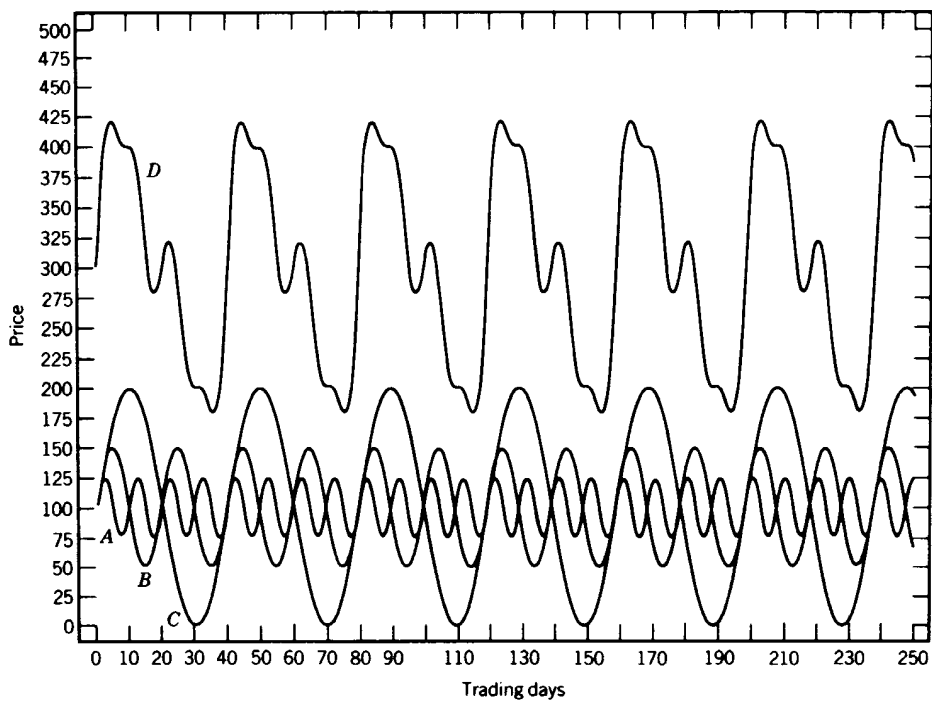
$$\hat{y}_t(\omega) = \frac{1}{\pi} \left(c_0 + 2 \sum_{k=1}^{N-1} c_k \cos \omega k \right)$$

where

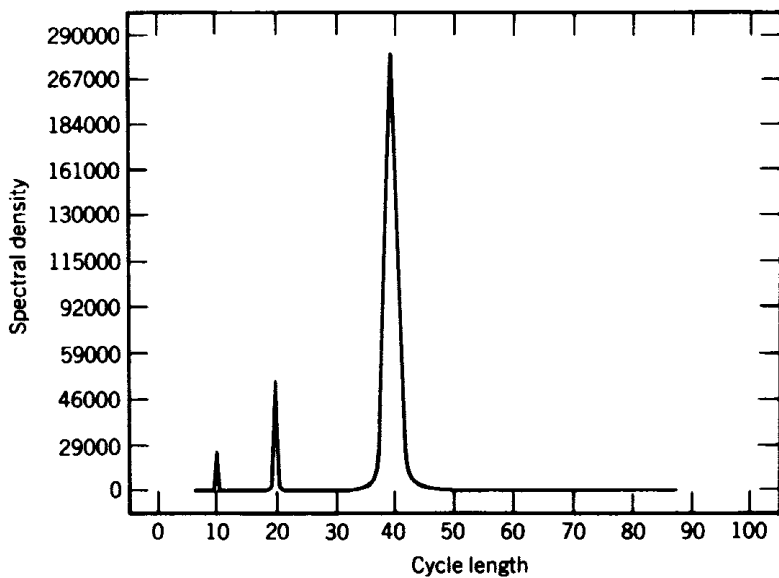
$$c_k = \sum_{t=1}^{N-k} \frac{(y_t - \bar{y})(y_{t+k} - \bar{y})}{N}$$

⁸ William T. Taylor, "Fourier Spectral Analysis," *Technical Analysis*

FIGURE 810 Spectral density (a) A compound wave D, formed from three primary waves, A, B, and C. (b) Spectral density of compound wave D.

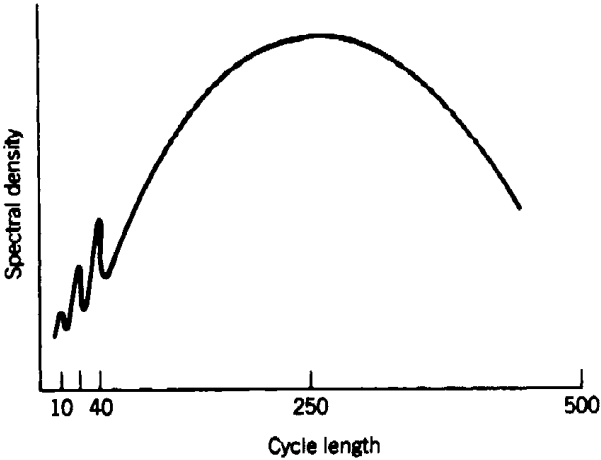


(a)



(b)

FIGURE 8-11 10-, 20-, and 40-day cycles, within a 250-day seasonal.



Methods of performing spectral analysis vary due to the choice that compensate for the fact that the accuracy of c_k decreases as k increases. Popular techniques for adjusting for this problem introduce an effective *window* and a truncation point $M < N$ so that the values of c_k for $k > M$ are not used and the values of c_k for $k \leq M$ are weighted by λ_k .

The spectral analysis approximation is then written:

$$\hat{y}_t(\omega) = \frac{1}{\pi} \left(\lambda_0 c_0 + 2 \sum_{k=1}^M \lambda_k c_k \cos \omega k \right)$$

where λ_k can be either of the following:

(a) *Tukey window*

$$\lambda_k = \frac{1}{2} \left(1 + \cos \frac{\pi k}{M} \right) \quad k = 0, 1, \dots, M$$

(b) *Parzen window*

$$\lambda_k = \begin{cases} 1 - 6 \left(\frac{k}{M} \right)^2 + 6 \left(\frac{k}{M} \right)^3 & 0 \leq k \leq \frac{M}{2} \\ 2 \left(1 - \frac{k}{M} \right)^3 & \frac{M}{2} \leq k \leq M \end{cases}$$

Using a Fast Fourier Transform Program

There are computer programs that apply a Fast Fourier Transform to perform a spectral analysis and create a Fourier power spectrum such as the one in Figure 810b. Anthony Warren's approach' can be found in Appendix 5 written in BASIC program code. The pro

`Anthony Warren,'A Mini Guide to Fourier Spectrum Analysis," Technical Analysis of Stocks & Commodities (January 1983). A very useful series of articles on spectral analysis has been published in Technical Analysis beginning in January 1983, authored by both Anthony W Warren and Jack K. Hutson. Much of the information in this section was drawn from that material.

gram detrends the data and reduces endpoint discontinuity, which can produce large unwanted cycles. This is accomplished by multiplying the data by a bellshaped window and extending the endpoints to give a more

definitive structure to the detrended data. without affecting the results (as discussed in the previous section).

A second filter is applied using selected moving averages. The moving average will reduce or eliminate the importance of those cycles, which are equal to or shorter than two times the length of the moving average, letting the more dominant cycles appear. For example, the use of a 10day moving average will eliminate cycles of length less than 20 days (frequencies greater than 12.5 per year). Figure 812 shows the output of the computer program.

Subsequent works by Warren and Hutson` present a computer program to calculate moving average weighted filters using linear, triangular, and Hanning weights.

Interpreting the Results of the Fourier Power Spectrum

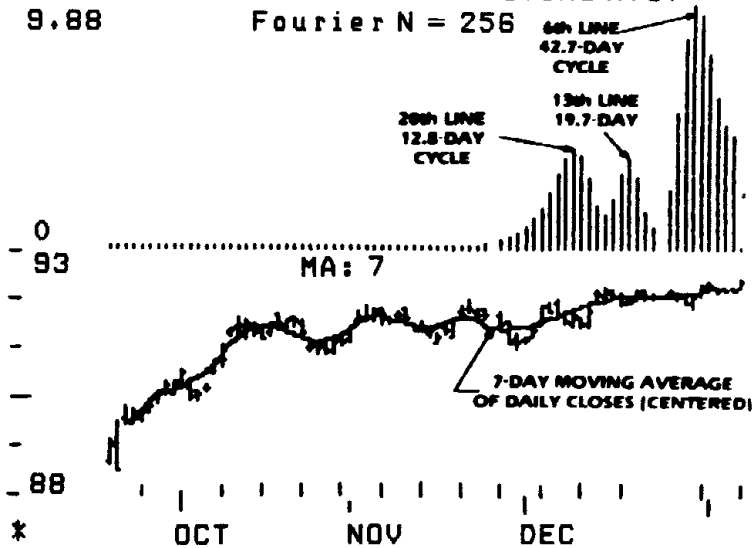
Both Figures 810b and 812 show a power spectrum resulting from a Fourier transform. Figure 810b is an ideal representation, in which the cycles stand out with no ambiguity; Figure 812 is more realistic, showing both the dominant cycles and a certain amount of variance around those values. In the power spectrum, the cycle power shown along the yaxis is the cycle amplitude squared. In Figure 810a, cycle D peaks at a price of about 425, which yields a spectral density, or spectral power, of 180,625 when squared. corresponding roughly to the 40day cycle in Figure 810b.

Using the information from the beginning of this chapter, the frequency is the inverse of the cycle length; therefore, if the cycle length is 40 days, the frequency $F = 360/40 = 9$.

` Anthony Warren and jack K. Hutson, "Finite Impulse Response Filter," Technical Analysis of Stocks & Commodities (May 1983).

FIGURE 812 Output of spectral analysis program.

TITLE T BILLS S03/83 01/07/83
 FOURIER ANALYSIS OF 7-DAY MOVING AVG.
 9.88 Fourier N = 256



Source.. jack K. Hutson, "Using Fourier," Technical Analysis of Stocks & Commodities, 1, no. 2 January & February 1983). @ 1983 Technical Analysis, Inc. Used with permission.

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The sine wave changes phase at the rate of 9 degrees per day, completing one fun cycle every 40 days.

A fast method for observing the possible results is to use weekly rather than daily data. This will be a close approximation for lowfrequency waves but will be less representative for the high frequencies. Averaging the data points can yield results very similar to the daily analysis.

MAXIMUM ENTROPY

Maximum Entropy Spectral Analysis (MESA) is a technique that filters noise (entropy) from a time series and exposes the useful cycles;" it provides a very practical alternative to Fourier analysis that makes it possible to find cycles using a very small amount of data. The use of Fourier transforms requires at least 256 data points and a minimum of 16 consistent cycles of 16 bars. That would eliminate the possibility of uncovering cycles for the shortterm trader.

John Ehlers describes the existence of shortterm cycles as a natural phenomenon. it is part of the process that causes rivers to meander back and forth as water seeks to flow in a straight line, or a drunkard who walks through an alley bumping against the walls but moving steadily forward. From these patterns, useful cycles can be found about 20% of the time. Using the Phase Angle

In an ideal situation, in which the market cycle can be shown as a pure sine wave, the phase angle constantly increases throughout the cycle,

beginning at 0 and ending at 360'. The phase angle then drops to 0 when the new cycle begins and increases again at a constant rate until it ends at 360'. This repeated pattern forms a sawtooth chart, as shown in Figure 813. Although the cycle goes from peak to value, the phase angle moves constantly in one direction.

in the practical analysis of shortterm cycles, Ehlers compresses tick data into bars of equal numbers of ticks, then examines the phase for uniformity. Once found, the uniform phase, which appears as a sawtooth chart, will become erratic as the shortterm cycle begins to break down, marking the end of a current market event.

Ehlers's Lateral Shift in Thinking

At first glance, the use of only a small amount of data needed by MESA seems to contradict the basic rules of statistics, which demand that results be based on as much data as possible to be reliable. But Ehlers, who has been the dominant influence in cycles since about 1990, is too knowledgeable to have made a mistake so simple. His book *MESA and Trading Market Cycles*⁶ focuses on the use of shortterm cycles based on short sample time periods. Instead, he has used this very attribute to apply cycles inside out.

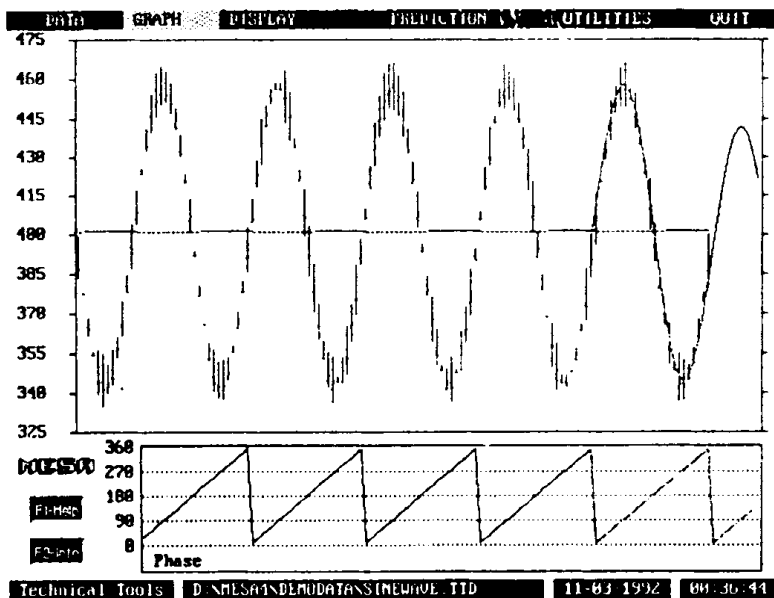
Ehlers's objective is to find very shortterm cycles. By definition, these cycles must be the result of human behavior, rather than based on market economics, because fundamentals are not usually relevant to periods of only a few days, and they are not likely to have a regular pattern when they make a rare appearance. If very shortterm cycles exist,

⁶ Anthony Warren, 'An Introduction to Maximum Entropy Method (MEM),' *TecAmical Analysis of Stocks & Commodities* (February 1984). See the bibliography for other articles on this topic.

⁷ John E Ehlers, "How to Use Maximum Entropy." *Technical Analysis of Stocks & Commodities* (November 1987).

⁸ John E Ehlers, *MESA and Trading Market Cycles* (John Wiley & Sons, New York, 1992).

FIGURE 813 The phase angle forms a sawtooth pattern.



Source: John Ehlers, "Cycle Analysis and Intraday Trading Technical Analysis of Stocks & Commodities, 11. no. 4 (February 1993). @ 1993 Technical Analysis, Inc. Used with permission.

they will not continue for long periods, and you must recognize them quickly if they are to be useful; therefore, shortterm cycles are found by analyzing only a small amount of recent data.

Then how does it help to find a shortterm cycle based on a small amount of data, if it is not statistically dependable? In a lateral shift, Ehlers uses the existence of a shortterm cycle to tell if prices are in a sideways pattern or trending. If a shortterm cycle exists, then the market trend should be weak. Ehlers has no interest in trading the cycle, which is surprising for a cycle expert, but prefers the dependability of the trend. He has, instead, attempted to solve one of the most difficult problems facing the analyst, trying to distinguish between a trending and sideways market. If a shortterm cycle exists, then we cannot rely on the trend. Ehlers develops this method throughout his book.

CYCLE CHANNEL INDEX

A trendfollowing system that operates for a market with a welldefined cyclic pattern should have specific qualities that do not necessarily exist in a generalized smoothing model. To confirm the cyclic turning points, which do not often occur precisely where they are expected, a standard moving average should be used, rather than an exponentially smoothed one. Although exponential smoothing always includes some residual effect of older data, the determination of a cyclic turning point must be limited to data that is nearer to one-fourth of the period, combined with a measure of the relative noise in the series, which may obscure the turn.

These features have been combined by Lambert" into a Commodity Cycle Index (M), which is calculated as follows:
 "Donald R. Lambert, "Commodity Channel Index Tools for Trading Cyclic Trends," Commodities (1980), reprinted in Technical Analysis of Stocks & Commodities.

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$$CCI_t = \frac{x_t - \bar{x}_t}{.015MD}$$

where $x_t = (H_t + L_t + C_t)/3$ is the average of the daily high, low, and close

$$\bar{x}_t = \sum_{i=t-N+1}^t x_i \text{ is the moving average over the past } N \text{ days}$$

$$MD = \sum_{i=t-N+1}^t |x_i - \bar{x}| \text{ is the mean deviation over the past } N \text{ days}$$

N is the number of days selected (less than $\frac{1}{4}$ cycle)

Because all terms are divided by N , that value has been omitted. In the CCI calculations, the use of $.015MD$ as a divisor scales the result so that 70% to 80% of the values fall within a +100 to -100 channel. The rules for using the CCI state that a value greater than +100 indicates a cyclic turn upward; a value lower than -100 defines a turn downward. Improvements in timing rest in the selection of N as short as possible but with a mean deviation calculation that is a consistent representation of the noise. The CCI concept of identifying cyclic turns is good because of the substantial latitude in the variance of peaks and valleys, even with regular cycles.

PHASING

One of the most interesting applications of the cyclic element of a time series is presented by J.M. Hurst in The Profit Magic of Stock Transaction Timing (PrenticeHall): it is the phasing or synchronization of a moving average to represent cycles. This section will highlight some of the concepts and present a simplified example of the method. It is already known that to isolate the cycle from the other elements, the trending and seasonal factors should be subtracted, reducing the resulting series to its cyclic and chance parts. In many cases, the seasonal and cyclic components are similar, but the trend is unique. Hurst treats the cyclic component as the dominant component of price movement and uses a moving average in a unique way to identify the combined trendcycle.

The system can be visualized as measuring the oscillation about a straightline approximation of the trend (centered line), anticipating equal moves above and below. Prices have many long and shortterm trends, depending on the interval of analysis. Because this technique was originally applied to stocks, most of the examples used by Hurst are longterm trends expressed in weeks. For commodities the same technique could be used b,,

applying the nearest futures contract on a continuous basis.

As a simple example of the concept, choose a moving average of medium length for the trending component. The fullspan moving average may be selected by, averaging the distance between the tops on a price chart. The halfspan moving average is then equal to half the days used in the fullspan average.

The problem with using moving averages is that they always lag. A 40day moving average is always 20 days behind the price movement. The current average is plotted under the most recent price, although it actually, represents the price pattern if the plot were lagged by onehalf the value of the average. This method applies a process called phasing, which aligns the tops and bottoms of the moving average with the corresponding tops and bottoms of the price movement. To phase the full and halfspan moving averages, lag each plot by half the days in the average; this causes the curve to overlay, the prices (Figure 814). Then project the phased fulland halfspan moving averages until they cross. A line or curve connecting two or more of the most recent intersections will be the major trendline. The more points used, the more complicated the regression formula for calculating

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the trend; Chapter 3 discusses a variety of linear and nonlinear techniques for finding the best fit for these intersections. Once the trendline is calculated, it is projected as the center of the next price cycle.

With the trend identified and projected, the next step is to reflect the cycle about the trend. When the phased halfspan average turns down at point A (Figure 815), measure the greatest distance D of the actual prices above the projected trendline. The system then anticipates the actual price crossing the trendline at point X and declining an equal distance D below the projected trendline. Once the projected crossing becomes an actual crossing, the distance D can be measured exactly and the price objective firmed. Rules for using this technique can be listed as follows:

1. Calculate the full-span moving average for the selected plot by half the days. If the full-span moving average is calculated at $t - F/2$, where t is the current time, then the phased average is calculated at $t - F/4$.
2. The half-span moving average is calculated for H days.
3. Record the points where the two phased averages intersect. The intersection points are $X_n, X_{n-1}, \dots$.
4. Find the trend by performing a linear regression on the intersection points $X_n, X_{n-1}, \dots$. If a straight line, then $Y_T = a + bX_T$.

FIGURE 814 Phasing.

5. Record the highest (or lowest) values of the price since t
6. Calculate the projection of the half-span by creating a st
est (or lowest) half-span value since the last crossing (A)
span value. This equation will be $Y_c = c + dX_c$.
7. Find the point at which the projected trendline crosses t
setting the equations equal to one another and solving f
crossing $(X_T, Y_T) = (X_c, Y_c)$, giving two equations in two v
solvable (X is time in days; Y is price).

8. If the halfspan is moving down, the maximum price reached by the commodity since the last crossing is subtracted from the Y coordinate of the projected crossing. This distance D is subtracted again from the Y coordinate to determine the price objective. If the halfspan is moving up, the price objective uses the minimum price and reflects the distance above the projected crossing. It should be noted that this calculation of distance is simplified because the trend is established by a straight line; for nonlinear fits, the measurement of D will be more complicated.

9. Recalculate the moving averages, the halfspan projection (6), the projected crossing (7), and price objective (8) each day until the actual crossing occurs. At that time D is fixed.

10. Follow the trading rules:

a. Enter a new long position when the halfspan moving average turns up; cover any existing short positions regardless of the price objective.

b. Enter a new short position when the halfspan moving average turns down; close out any long positions.

c. Close out both long and short positions if the price objective is reached. An allowable error factor is considered as 10% of the height of the fun cycle (lowest to highest point).

This approach to cycles should be studied carefully as an example of a complex problem solved using elementary mathematics. There are many techniques for determining trends and a number of seasonally oriented systems, but a cyclic approach is rare. Whereas Hurst's explanation is more complete and more sophisticated, the interpretation presented in this section should be considered only a reasonable approximation.